Chomsky Normal Form

Definition

Chomsky Normal Form means that every rule in the grammar is either of the form $A \to a$ or $A \to BC$.

Any context free grammar can be converted to a grammar in Chomsky Normal Form (CNF) via a sequence of steps described below.

To follow along either enter the following rules into JFLAP or upload the file CNF.jff.

$$
\begin{aligned}
&S \to aAbA \\
&S \to SA \\
&A \to \varepsilon \\
&A \to Y \\
&Y \to bY \\
&Y \to b
\end{aligned}
$$

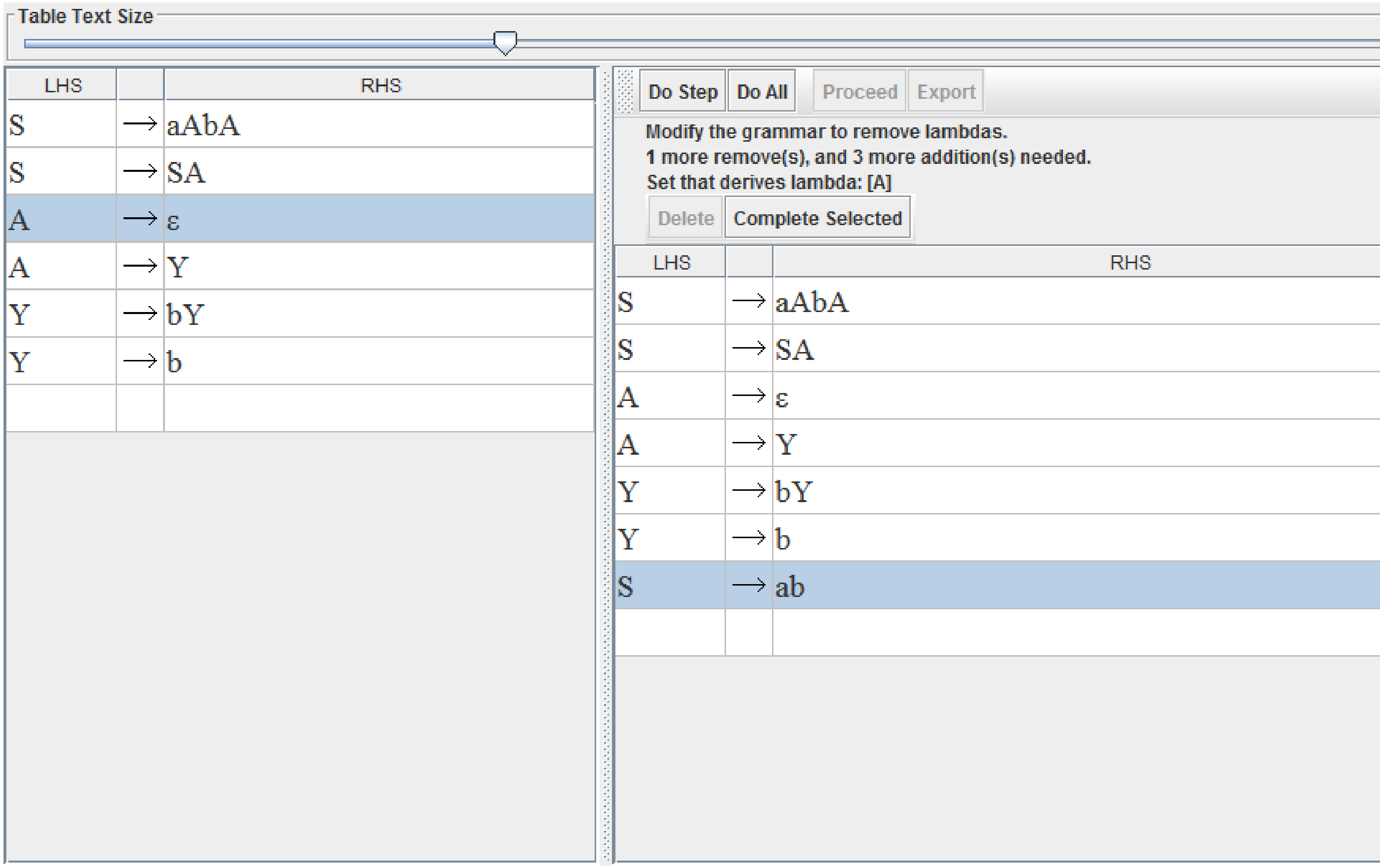
Eliminating epsilon rules

The first step to converting a grammar into CNF is to remove the rules that produce an empty string. In this case there is only one of them $A \to \varepsilon$.

To remove one of these types of rules we basically have to add extra rules that simulate the effect of replacing the variable in question with an ε. For instance $S \to aAbA$, if we replace the first A we get $S \to abA$. replace the second A to get $S \to aAb$ and replace both As with ε to get $S \to ab$. It is important while doing the conversion that every single possibility is added as a rule. We do not want to have any negative effects of eliminating the ε rule.

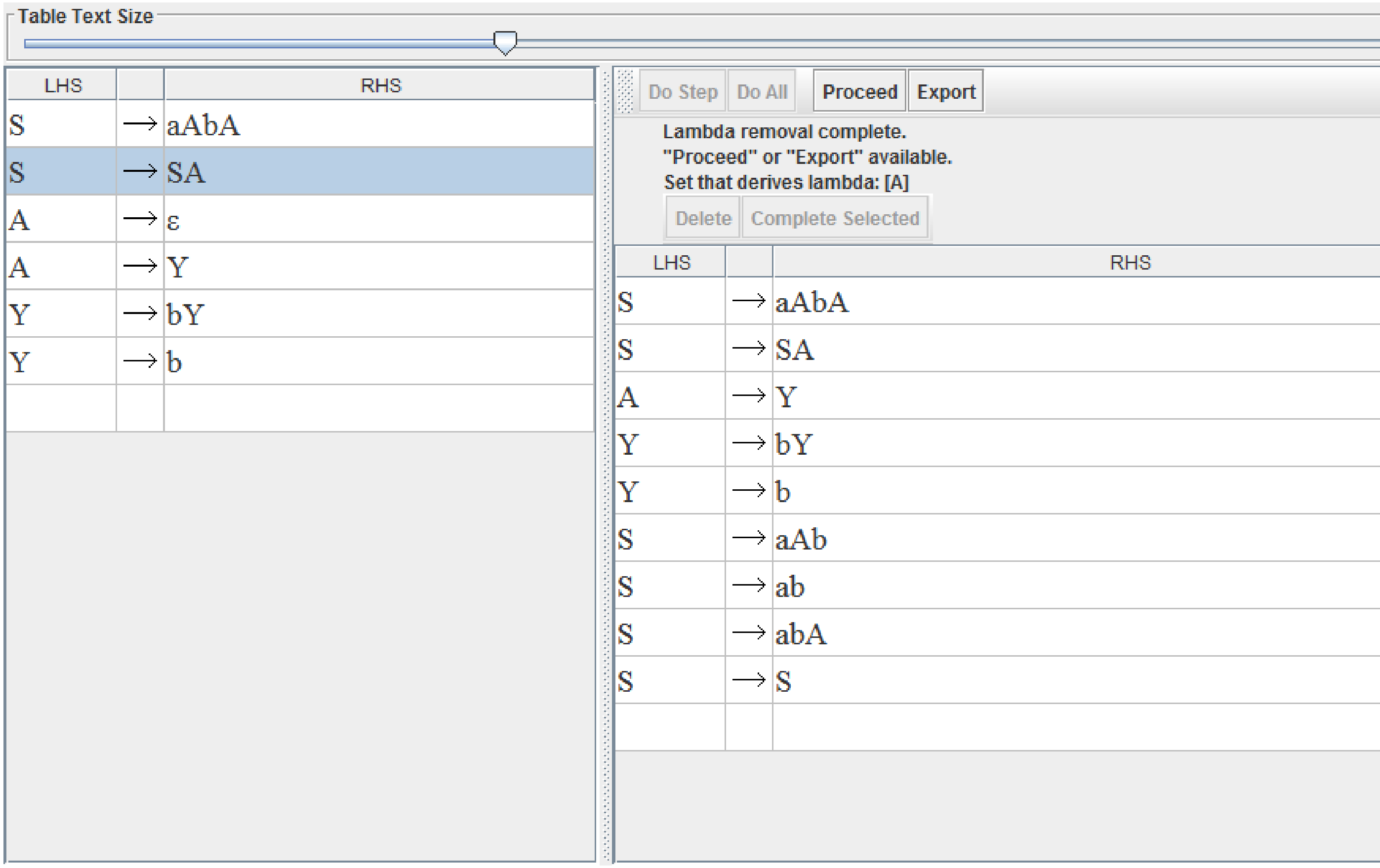
In JFLAP once you have the grammar loaded up just click on the Transform Grammar option. The first step corresponds to Lambda removal. JFLAP will ask you to click the ε rules and then ask you to remove them one by one. The convenience with JFLAP is that it keeps track of how many rules need to be removed and how many need to be added.

For instance if we just add the $S \to ab$ rule we get the following message.



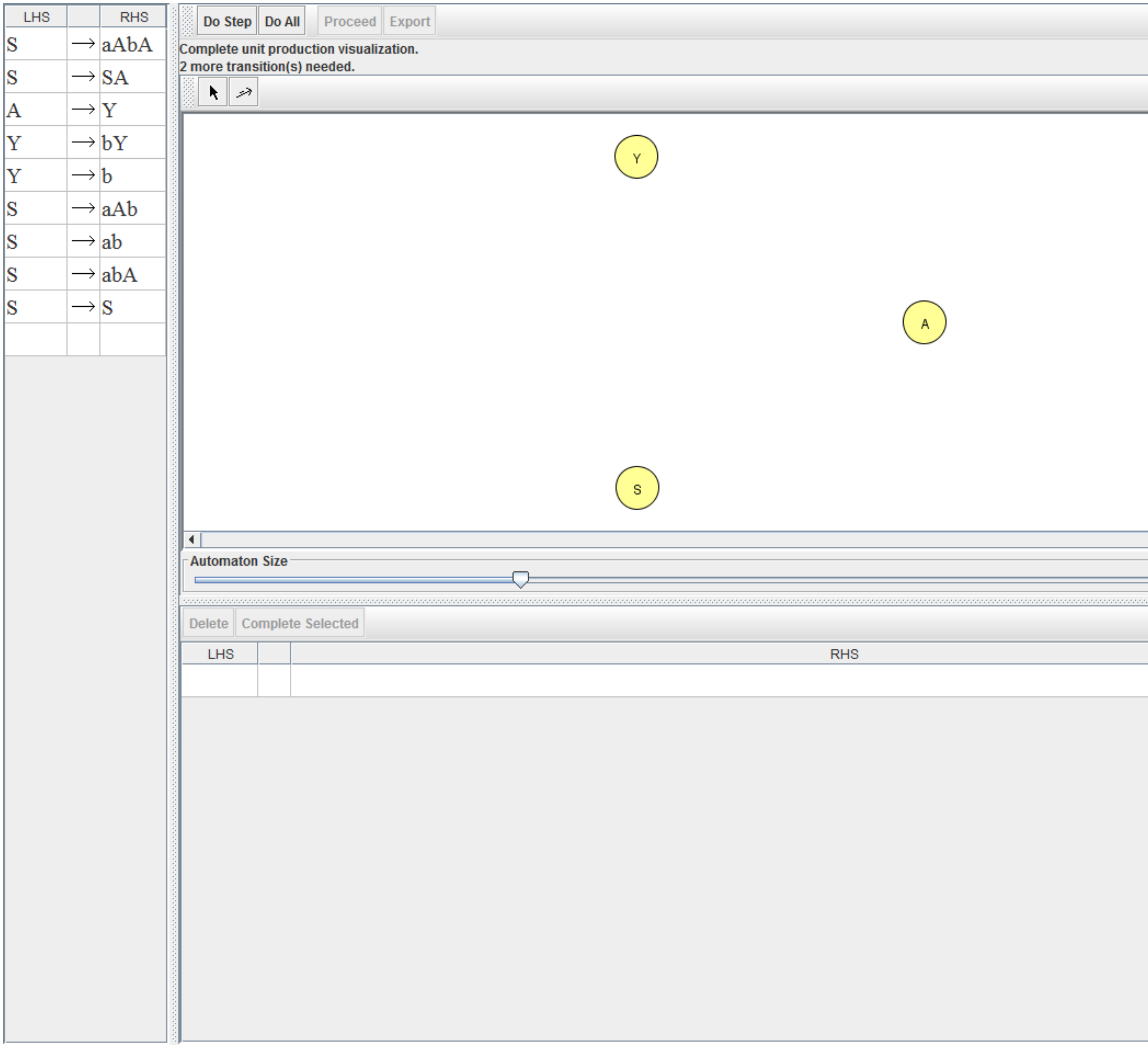
If you make a mistake you will be informed of it by JFLAP saying it is not part of the new grammar.

and here is the complete picture after eliminating the single ε rule

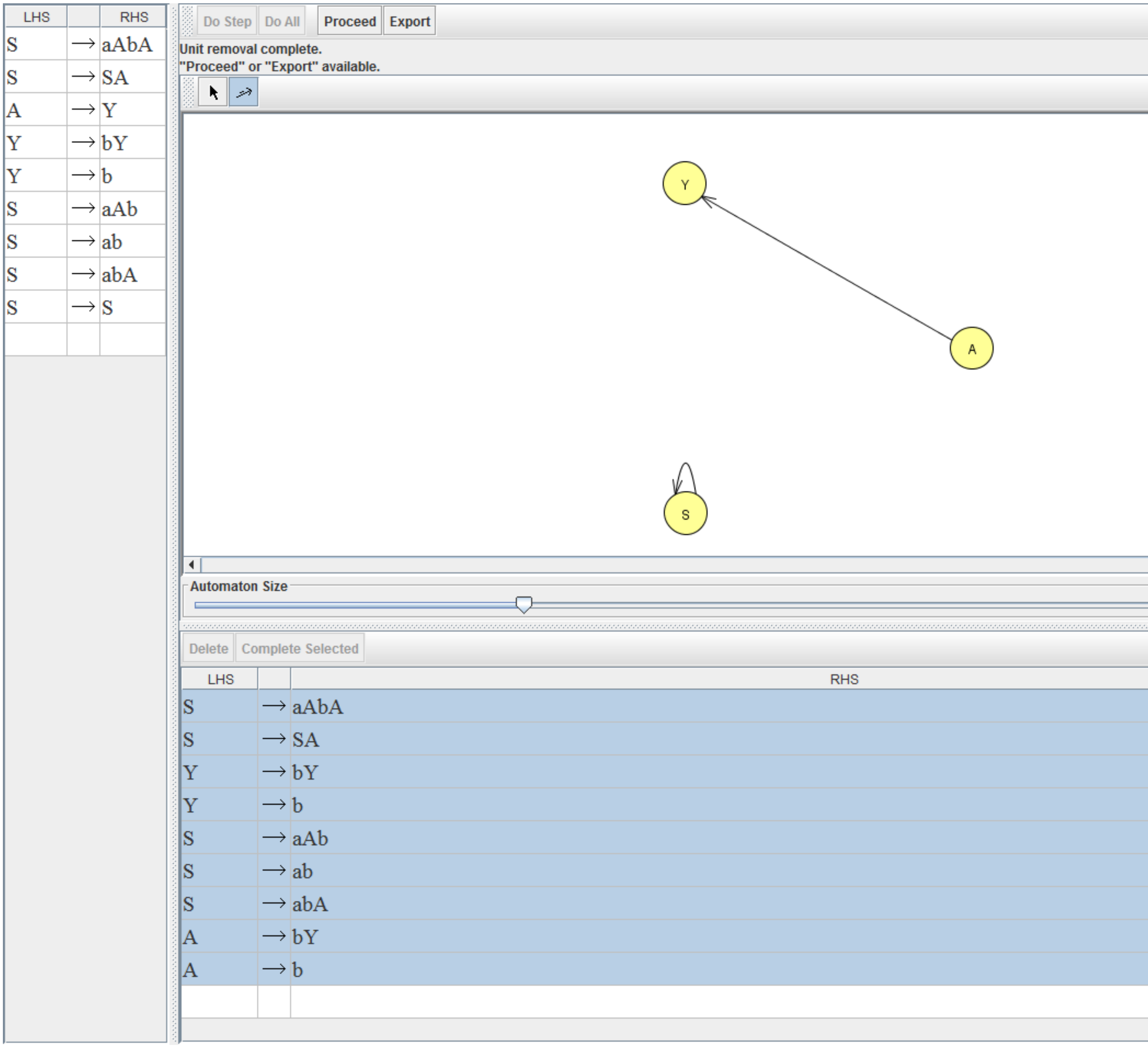


Eliminating unit rules

The next step is eliminating unit rules, rules where one non-terminal just produces another non-terminal. JFLAP makes this easy by basically allowing you to construct a relationship graph.



Connect up the nodes as per the unit rules.



Removing $A \to Y$. The same idea as removing the ε is used. The effect of having that rule $A \to Y$ is that you get from A to anything that Y can produce. So instead of having to go through this transition we just directly make A go to the things that Y can produce. So extra rules

$$A \to b$$
$$A \to bY$$

have to be added if we want to remove $A \to Y$.

there is still one unit rule $S \to S$ but that can immediately be removed since it is a useless rule.

This results in the following set of rules.

$$
\begin{aligned}
S &\to aAbA \\
S &\to SA \\
Y &\to bY \\
Y &\to b \\
S &\to aAb \\
S &\to ab \\
S &\to abA \\
A &\to bY \\
A &\to b
\end{aligned}
$$

Adding extra variables

The last step is adding extra variables so that every rule is as per the CNF specifications. This means rules like $S \to aAbA$ will have to be converted over by adding additional variables. This can be done one at a time from the left as follows.

$$
\begin{aligned}
S &\to B(a)D(1) \\
D(1) &\to AD(2) \\
D(2) &\to B(b)A \\
B(a) &\to a \\
B(b) &\to b
\end{aligned}
$$

In general we replace each rule $A \to u_1u_2 \ldots u_k$, where $k \geq 3$ and each u_i is a terminal or non-terminal with the rules $A \to u_1A_1$, $A_1 \to u_2A_2$, $A_2 \to u_3A_3, \ldots,$ and $A_{k-2} \to u_{k-1}u_k$. The A_i are new variables being introduced. We also need to replace any terminal u_i with $U_i \to u_i$.

The final set of rules look like this

LHS		RHS
S	$\rightarrow$	aAbA
S	$\rightarrow$	SA
S	$\rightarrow$	aAb
S	$\rightarrow$	ab
S	$\rightarrow$	abA
A	$\rightarrow$	b
A	$\rightarrow$	bY
Y	$\rightarrow$	b
Y	$\rightarrow$	bY

Convert Selected | Do All | What's Left? | Export

All productions completed.
Conversion done. Press "Export" to use.

LHS		
S	$\rightarrow$	B(a)D(1)
D(1)	$\rightarrow$	AD(2)
B(a)	$\rightarrow$	a
B(b)	$\rightarrow$	b
S	$\rightarrow$	SA
S	$\rightarrow$	B(a)D(3)
D(3)	$\rightarrow$	AB(b)
S	$\rightarrow$	B(a)B(b)
S	$\rightarrow$	B(a)D(2)
D(2)	$\rightarrow$	B(b)A
A	$\rightarrow$	b
A	$\rightarrow$	B(b)Y
Y	$\rightarrow$	b
Y	$\rightarrow$	B(b)Y